$$\text{Si} \quad \psi = 0{,}5\,\psi_1 + C\psi_2$$

$$\psi_1 = \psi_{100}(r) = \frac{1}{\sqrt{\pi a^3}}\, e^{-\frac{r}{a}}$$

$$\psi_2 = \psi_{21-1}(r) = -i\,\frac{r\sin\theta}{8a\sqrt{\pi a^3}}\exp\left(-\frac{r}{2a}\right)\exp(i\varphi)$$

$$\psi = \frac{1}{\sqrt{\pi a^3}}\left(\frac{1}{2}\exp\left(-\frac{r}{a}\right) - \frac{irC\sin\theta}{8a}\exp\left(-\frac{r}{2a}\right)\exp(i\varphi)\right)$$

$$\langle r\rangle = \int \psi^*(r)\, r\, \psi(r)\, dr$$

$$\langle r^2\rangle = \int \psi^* r\, r^2 \psi(r)\, dr =$$

$$= \frac{1}{\sqrt{\pi a^3}}\int\left(\frac{1}{2}e^{-\frac{r}{a}} + \frac{irC\sin\theta}{8a}\exp^{-\frac{r}{2a}}\cos\varphi + \frac{rC\sin\theta}{8a}e^{-\frac{r}{a}}\sin\varphi\right)\cdot$$

$$\cdot\, r^2\cdot\left(\frac{1}{2}e^{-\frac{r}{a}} + \frac{rC\sin\theta}{8a}e^{-\frac{r}{2a}}(i\cos\varphi + \sin\varphi)\right)dr =$$

$$= \int r^2\left(\left(\frac{1}{2}e^{-\frac{r}{a}} + \frac{rC\sin\theta}{8a}e^{-\frac{r}{2a}}\sin\varphi\right)^2 + \left(\frac{rC\sin\theta}{8a}\right)^2 e^{-\frac{r}{a}}\cos^2\varphi\right)dr$$



$$= \int r^2 \left(\frac{1}{4} e^{-\frac{2r}{a}} + \left(\frac{r c \sin\theta}{8a} e^{-\frac{r}{2a}} \right)^2 + \frac{r c \sin\theta}{8a} e^{-\frac{3r}{2a}} \right) dr$$

Далі р-к за інт. Пуассона

№2 $L = \left(\frac{d}{dx} e^{2x}\right)^2$

На ф-ю $\sin x$

$$\left(\frac{d}{dx} e^{2x}\right)^2 \sin x = \left(\frac{d}{dx} e^{2x}\right)\left(\frac{d}{dx} e^{2x}\right)\sin x =$$

$$= \left(\frac{d}{dx} e^{2x}\right) \frac{d}{dx}\left[\sin x \cdot e^{2x}\right] =$$

$$= \left(\frac{d}{dx} e^{2x}\right)\left[2e^{2x} \cdot \sin x + e^{2x}\cos x\right] =$$

$$= \left(\frac{d}{dx} e^{2x}\right) e^{2x}\left[2\sin x + \cos x\right] =$$

$$= 7e^{4x}\sin x + 6e^{4x}\cos x$$

№3 $\hat{A} = -i\hbar\frac{\partial}{\partial\varphi} + \lambda\sin\varphi$

$\hat{A}\psi = -i\hbar\frac{\partial\psi}{\partial\varphi} + \lambda\sin\varphi\cdot\psi = \lambda\psi$

$i\hbar\frac{\partial\psi}{\partial\varphi} = \psi(\lambda\sin\varphi - \lambda) \Rightarrow$

$\Rightarrow i\hbar\frac{\partial\psi}{\psi} = (\lambda\sin\varphi - \lambda)\partial\varphi \Rightarrow$

$\Rightarrow \frac{\partial\psi}{\psi} = \frac{(\lambda\sin\varphi - \lambda)}{i\hbar}\partial\varphi$

$\ln\psi = \frac{1}{i\hbar}(-\lambda\cos\varphi - \lambda\varphi)$

$\psi = \exp\left[-\frac{1}{i\hbar}(\lambda\varphi + \lambda\cos\varphi)\right]$

$\lambda_m = \hbar m$

$\psi_m = \frac{\exp\left[i\left(m\varphi + \lambda\cos\frac{\varphi}{\hbar}\right)\right]}{\sqrt{2\pi}}$